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Application of a New Time Scale Based Low K-ε Model to Natural Convection from a Semi-Infinite Vertical Isothermal Plate

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AMARILLO NATIONAL RESOURCE CENTER FOR PLUTONIUM/
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A Report on

Application of a New Time Scale Based Low K-ε Model To Natural Convection from a Semi-Infinite Vertical Isothermal Plate

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Abstract

The low k-ε model proposed by Yang and Shih (1992) is applied to the calculation of the turbulent natural convective boundary layer over a semi-infinite, vertical, isothermal surface. Using k/ε as the turbulent time scale will introduce a singularity in the ε equation, near the wall. This model uses a modified turbulent time scale near the wall to eliminate this singularity. The constants in the equation for damping function are modified to produce better results for both, natural convection and forced convection. The results are compared with available experimental data and the results obtained from Chien's model and are found to be in reasonable agreement.

1. INTRODUCTION

Heat transfers by natural convection from vertical isothermal surfaces have received considerable research interest since these flows are encountered in many industrial applications. The two equation k-ε model is widely used to model turbulent flows. Here k represents the turbulent kinetic energy and ε represents the dissipation rate of turbulent kinetic energy. This model is usually applied with logarithmic wall functions for velocity and temperature, to avoid the application of the k-ε model in the laminar sub-layer near the wall. These wall functions only hold for forced convection flows; they do not hold for natural convection flows. Proper wall functions for natural convection flows have still not been found. Therefore low k-ε model should be applied for natural convection flows, in which the calculations are performed up to the wall. The first low k-ε model was proposed by Jones and Launder, which was then followed by a number of other models.

Yang and Shih proposed a new time scale based k-ε model for near wall turbulence. In this model, the eddy viscosity is characterized by a turbulent velocity scale and a turbulent time scale. It uses a modified time scale near the wall such that there is no singularity in the dissipation equation near the wall. The model constants are exactly the same as those in the standard k-ε model that ensures the performance of the model far from the wall. In this paper the constants in the damping function are modified to provide better results for both, natural convection and forced convection. The modified model is applied to the calculation of the turbulent natural convective boundary layer over a vertical, isothermal surface. The results are compared with available experimental data and the results obtained from Chien's model.

2. MODEL EQUATIONS

The time averaged turbulent boundary layer equations for two-dimensional, incompressible buoyancy induced flow are as follows.

Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\frac{\partial}{\partial y}$$
$$(\nu+\nu_t)\frac{\partial u}{\partial y}+g\beta(t-t_\infty) \qquad (2)$$

Energy:

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\nu}{\mathrm{Pr}}+\frac{\nu_t}{\mathrm{Pr}_t}\right)\frac{\partial t}{\partial y} \qquad (3)$$

It is assumed that the Boussinesq is valid and

$$u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial y}=\frac{\partial}{\partial y}\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}+\nu_t\left(\frac{\partial u}{\partial y}\right)^2-\varepsilon \qquad (4)$$

the turbulent stresses are proportional to the mean velocity gradients.

$$u\frac{\partial \varepsilon}{\partial x}+v\frac{\partial \varepsilon}{\partial y}=\frac{\partial}{\partial y}\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}$$
$$+\left[c_{1\varepsilon}\nu_t\left(\frac{\partial u}{\partial y}\right)^2-c_{2\varepsilon}\varepsilon\right]/T_t \qquad (5)$$

The transport equations for k and ε are given as follows.

k equation:

ε equation:

The time scale (T_t) used in the standard k-ε model is k/ε. Using this time scale up to the wall will introduce a singularity in the ε equation due to vanishing k at the wall. Therefore near the wall this time scale has to be modified to avoid this singularity. Yang and Shih showed that the time scale near the wall should be the Kolmogorov time scale because viscous dissipation dominates near the wall. Therefore they proposed that the turbulent time scale be given by k/ε away from the wall and by the Kolmogorov time scale near the wall. The time scale for the whole region could be written as

$$T_t = k/\varepsilon + T_k \qquad (6)$$

where T_k is the Kolmogorov time scale and is given by

$$T_k = c_k\,(\nu/\varepsilon)^{1/2} \qquad (7)$$

This time scale ensures the performance of the standard k-ε model far from the wall since k/ε is much larger than T_k away from the wall and the model constants are the same as those in the standard k-ε model. Since k/ε vanishes near the wall due to the boundary condition for k, the time scale near the wall would be T_k.

Using this time scale as the turbulent time scale and $k^{1/2}$ as the turbulent velocity scale would give the below expression for the eddy viscosity.

$$\nu_t = c_\mu\, f_\mu\, kT_t \qquad (8)$$

f_μ is the damping function, which is used to account for the wall effect and is given by

$$f_\mu = [1-\exp(-a_1R_y - a_3R_y^{\,3} - a_5R_y^{\,5})]^{1/2} \qquad (9)$$

where

$R_y = k^{1/2} y/\nu$ (10)

and $a_1 = 3.0*10^{-4}$, $a_2 = 6.0*10^{-5}$, $a_3 = 2.0*10^{-6}$.

The suggested boundary condition for ε on the wall is:

$\varepsilon_w = 2\nu(dk^{1/2}/dy)^2$ (11)

3. NUMERICAL SOLUTION

The free convection turbulent boundary layer equations are solved for a semi-infinite isothermal flat plate. Air is the fluid considered for these calculations. The EXPRESS code, which uses the standard k-ε model and the Reynolds stress model, is modified with the above low k-ε model and used to for these calculations. The above governing equations are transformed from the physical coordinates (x, y) into the coordinate system (x,η) where the non-dimensional cross stream coordinate η is defined as:

$$\eta = y/\delta. \qquad (12)$$

Even though δ varies with x, the computational domain is constrained to lie in the region:

$$0 \leq \eta \leq 1. \qquad (13)$$

The control volume approach is used to discretize the governing equation. In this approach the governing partial differential equations are converted into algebraic equations by integrating them over the cells.

Since for gases and liquids with moderate Prandtl number transition to turbulent flow occurs between $Gr_x = 10^9$ and 10^{10}, the calculation was started at $Gr_x = 10^{10}$. Profile shapes used by Eckert and Jackson are used for initial velocity and temperature profiles. These profiles are given by:

$$u/U = (y/\delta)^{1/7} (1 - y/\delta)^4 \qquad (14)$$

$$(t - t_\infty)/(t_w - t_\infty) = 1 - (y/\delta)^{1/7} \qquad (15)$$

where U is the characteristic velocity and is given by:

$$U = [g\beta(t_w - t_\infty)x]^{1/2} . \qquad (16)$$

At the free stream, the velocity components are set to zero, the turbulence quantities are set to a pre-set free stream values and the temperature is set to a constant value (t_∞). At the wall the velocity components are set to zero, the temperature is set to a constant value (t_w), k is set to zero and ε is given by Equation (11).

At each x station, the system of non-linear equations are solved using the Thomas algorithm (TDMA) and then marched downstream to the next x station.

4. RESULTS AND DISCUSSIONS

The results obtained from the above model are compared with the available experimental results and the results obtained by using Chien's low k-ε model. Calculations are performed for Pr = 0.71 with 44 grids in the y direction. The non-dimensional turbulent velocity and temperature profiles at $Gr_x = 10^{12}$ are compared with the profiles obtained from Chien's model in Figure 1 and Figure 2 respectively. These profiles are plotted versus η. The velocity is normalized with the characteristic velocity U. The profiles are agreeing well with those obtained using Chien's model.

The distribution of wall shear stress τ_w is shown in Figure 3. From the experimental results Tsuji and Nagano found a correlation for the turbulent shear stress. It is given by

$$\tau_w / \rho U_b^2 = 0.684\, Gr_x^{1/11.9} \qquad (17)$$

The shear stress distribution is compared with the above correlation and the distribution obtained from Chien's model and found to be agreeing well.

Figure 4 shows the wall heat transfer as a function of Gr_x. The calculated values are compared with the values obtained from Chien's model and the best fit curve found from the experimental values. The best fit curve for Nu_x for air at large Gr_x is given by

$$Nu_x = 0.106\, Gr_x^{1/3} \qquad (18)$$

Though the wall heat transfer calculated using Chien's model is close to the experimental values at the beginning, it is becoming too high for large Gr_x. The presented low k-ε model is under predicting the wall heat transfer. But it is becoming close to the experimental results with increasing Gr_x. The difference at the beginning may be due to the influence of the prescribed initial conditions. At large Gr_x, results become independent of the initial profiles and the model gives better results.

5. CONCLUSIONS

The low k-ε model proposed by Yang and Shih is modified and applied to calculate turbulent buoyancy driven flow over a semi-infinite plate. By comparing the results with experimental results and results from Chien's model, it is shown that the model gives results with reasonable accuracy. Better results could be obtained by further refinement of the model.

Since the model constants used in this model are exactly the same as those in the standard k-ε model, away from the wall it reduces to the standard k-ε model. Therefore this model could be used for both, the near wall turbulence and high Reynolds number turbulence.

NOMENCLATURE

a_1, a_2, a_3	Constants in the equation for damping function
c_k	Constant in the equation for Kolmogorov time scale, 1.0
$c_{1\varepsilon}$, $c_{2\varepsilon}$	Model constants
c_μ	Coefficient in the equation for ν_t
f_μ	Damping function
g	Gravitational acceleration
Gr_x	Local Grashof number, $g\beta(t_w-t_\infty)x^3/\nu^2$
k	Turbulent kinetic energy
Nu_x	Local Nusselt number
p	Pressure
Pr	Prandtl number
Pr_t	Turbulent Prandtl number for t
T_t	Turbulent time scale
T_k	Kolmogorov time scale
t	Temperature
t_w	Wall temperature
t_∞	Free stream temperature
U	Characteristic velocity
U_b	Normalizing velocity, $(g\beta(t_w- t_\infty)\nu)^{1/3}$
u	Vertical velocity component
v	Velocity component perpendicular to the plate
x	Vertical coordinate
y	Horizontal coordinate
β	Coefficient of thermal expansion
ε	Dissipation of kinetic energy
ρ	Density
τ	Shear stress
σ_k	Turbulent Prandtl number for k
σ_ε	Turbulent Prandtl number for ε
ν	Kinematic viscosity
ν_t	Turbulent kinematic viscosity

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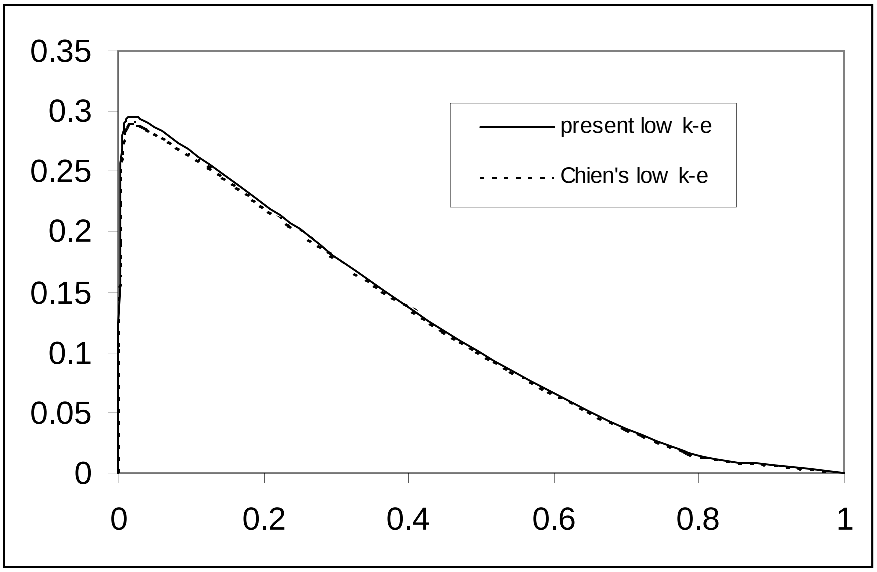

Figure 1: Velocity Profile at $Gr_x = 10^{12}$ (Variation of u/U with η)

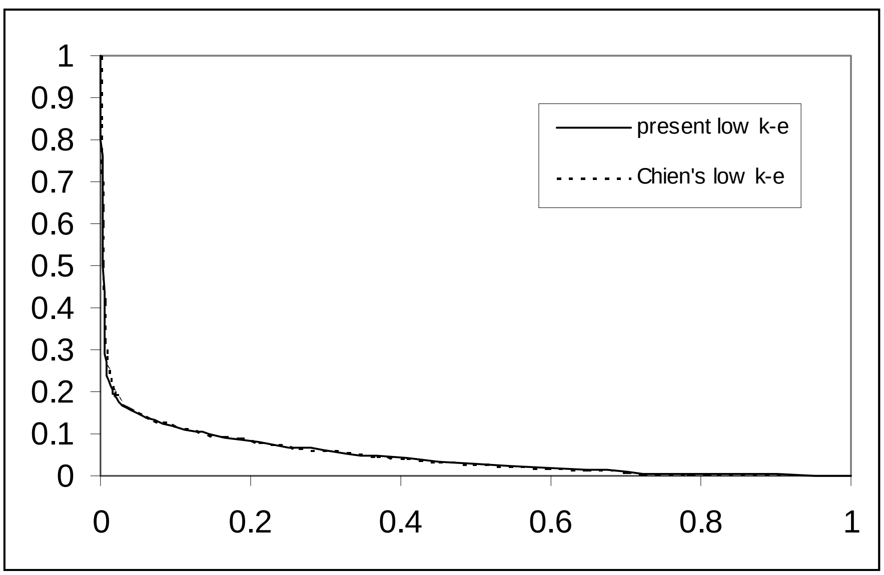

Figure 2: Temperature Profile at $Gr_x = 10^{12}$ (Variation of $(t-t_\infty)/(t_w - t_\infty)$ with η)

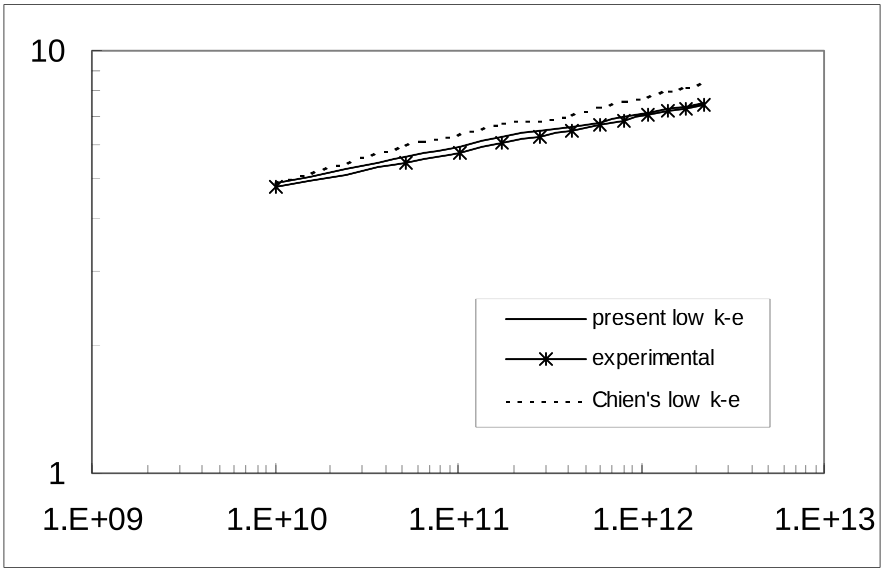

Figure 3: Wall Shear Stress (Variation of $\tau_w/\rho U_b^2$ with Gr_x)

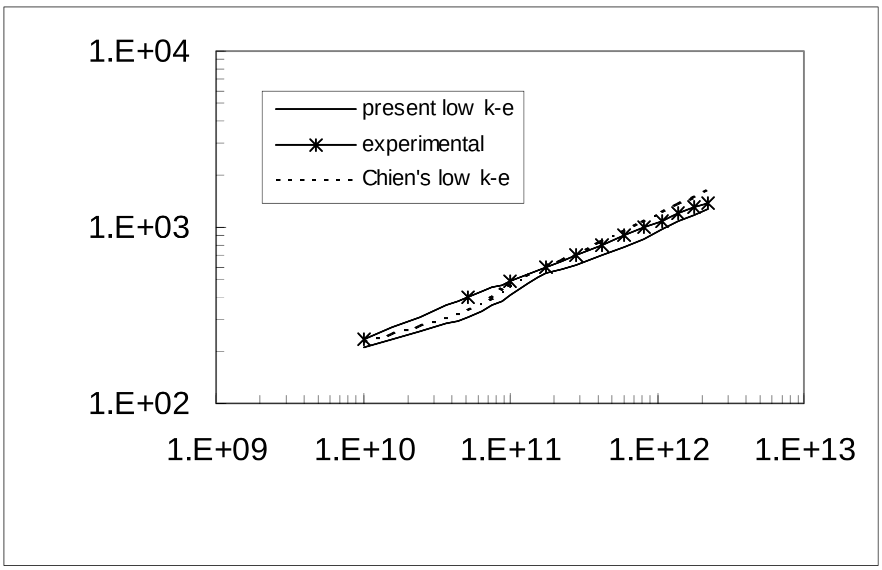

Figure 4: Wall Heat Transfer (Variation of Nu_x with Gr_x)